Wavefront Sensing

Alan Greenaway

Credit to:

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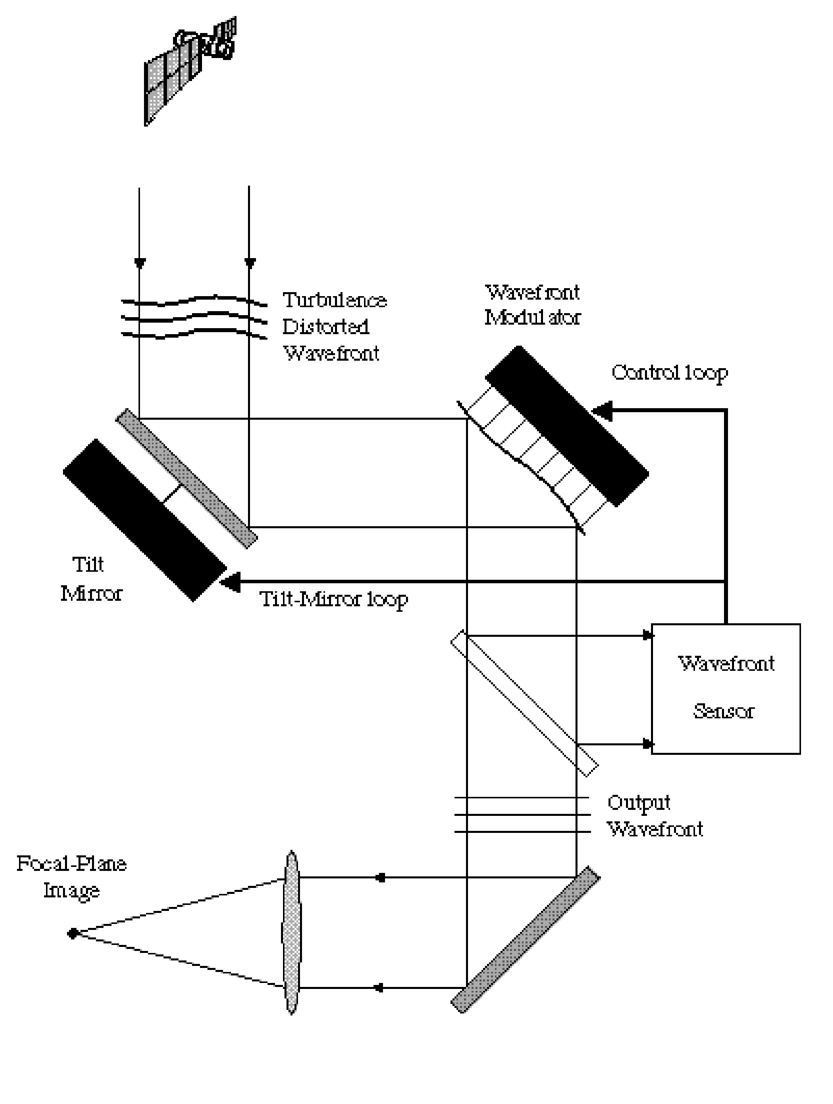
Adaptive Optics - what is it?



- **Adaptive = feedback control**
- Adaptive Optics
  - 3 Components
    - Wavefront Modulator (WFM)
    - Wavefront Sensor (WFS)
    - Control loop
- Active optics
  - No WFS
  - No on-line control loop
  - Control signal pre-computed off-line (e.g. gravity sag, thermally-induced aberrations, ...)

Background

- Adaptive optics
  - Null wavefront sensor is most suitable
    - faster
    - flexible
    - cheaper (?)
- Metrology
  - Require wavefront reconstruction
    - reference to national standards
    - speed/cost less important (?)

Metrology

- Metrology is a demanding application
  - Numbers
    - Even worse, absolute numbers!
    - Reference to standards!
- Speed also required...
  - Production lines for thin plastic films run at $5ms^{-1}$
- …as is robustness
  - Discontinuous surfaces
  - Rough surfaces
  - Multiply-connected surfaces

Err...

- For industrial surfaces
  - ➢ Sub nm resolution is generally sufficient
- For thin films
  - ➢ Sub Å resolution on wavefront aberrations is required
- For exo-earth imaging
  - ➢ Better than 0.1 Å is required

An industrialist insisted that he needed a resolution of $\lambda/40000$

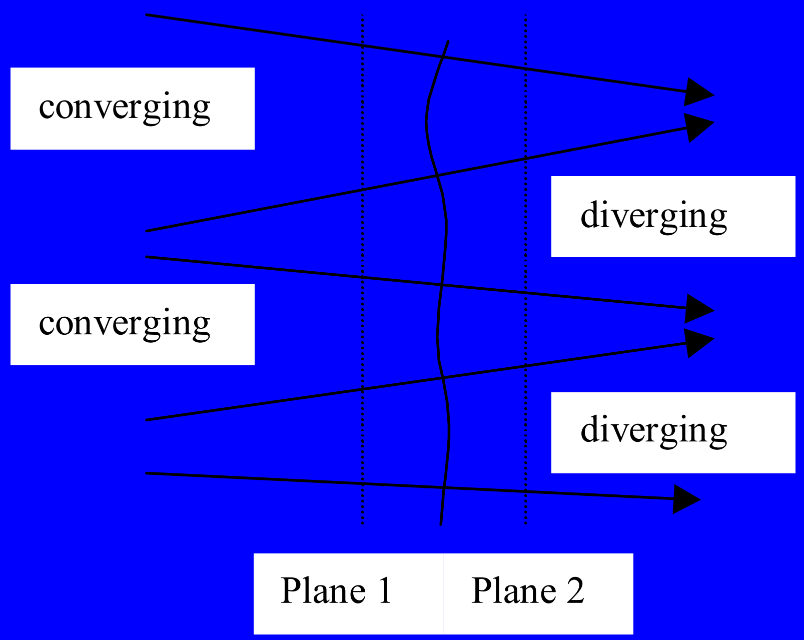
How to achieve these...

- Average over lenslet (Shack Hartmann)?
- Shearing interferometer?
- Wavefront curvature (phase diversity)?

- ➢ Unlikely to give accuracy or spatial format needed
- ➢ Data reduction is time consuming
- ➢ Efficient use of data (not of detector)
- ➢ Can be reduced to a very simple algorithm
- ➢ Can be very accurate
- ➢ Can be large format

Phase-diverse wavefront sensing

(wavefront curvature sensing)



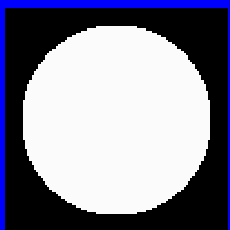
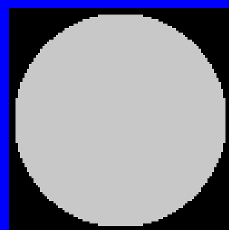
- Solution of ITE gives wavefront

$$\Psi(r) = -k\int_R dr' G(r,r')\frac{\partial I(r')}{\partial z}$$

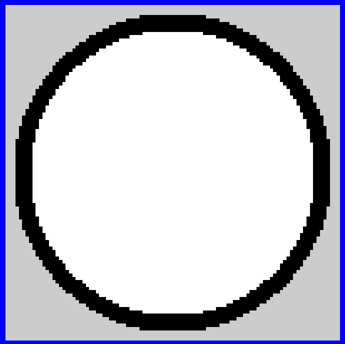
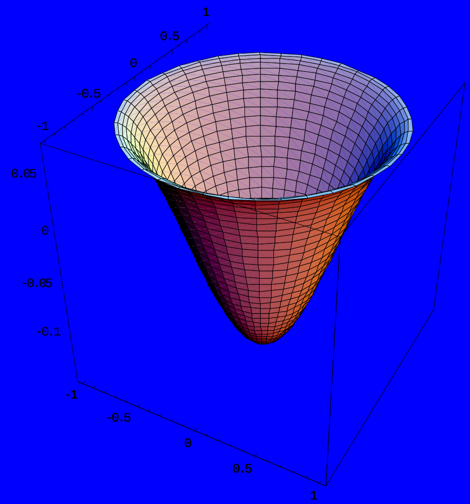
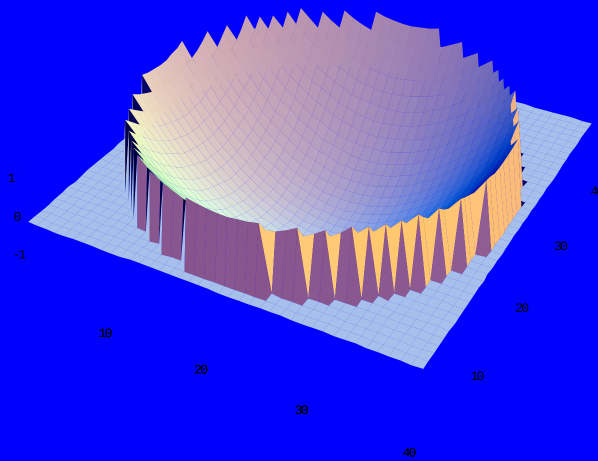
$$\frac{I_{\text{Plane 1}} - I_{\text{Plane 2}}}{z_1 - z_2} \approx \frac{\partial I}{\partial z}$$

Green's function solution (GFS)

I_1 I_2

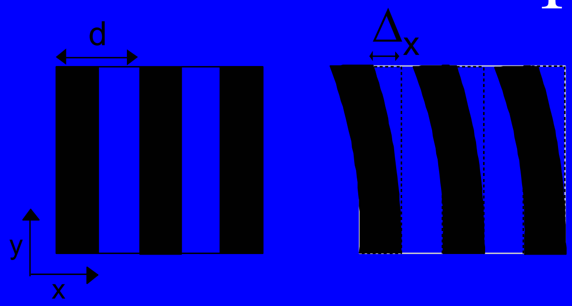
 - 

$$\frac{2\pi}{\lambda}\cdot\frac{(I_1-I_2)}{\bar{I}\,\delta z}\cdot G=\phi$$

 •  = 

How to get the data?

- Use a diffraction grating as a beamsplitter
- Use 'Detour Phase' to get different level of defocus in each diffraction order.
- Simultaneous multi-plane imaging on 1 CCD



$$\text{LocalPhaseShift } \phi_m = 2\pi\, m \frac{\Delta_x}{d}$$

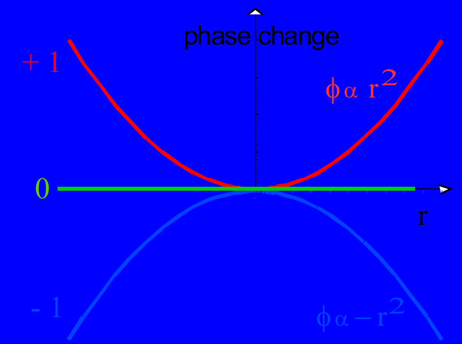
Phase shift is of opposite sign
in orders of opposite sign m.

How to collect data?

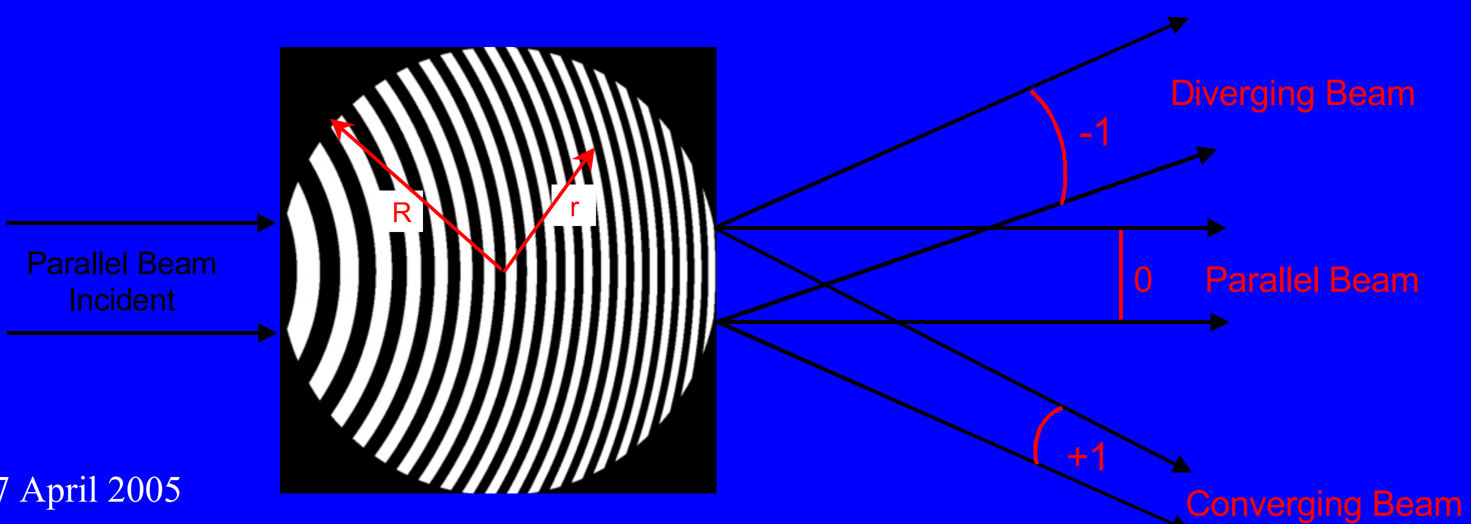
W_{20} defines level of distortion. Equivalent to extra path length introduced at edge of grating aperture

Distortion

$$\Delta(x,y) = \frac{W_{20}d}{\lambda R^2}(x^2+y^2)$$



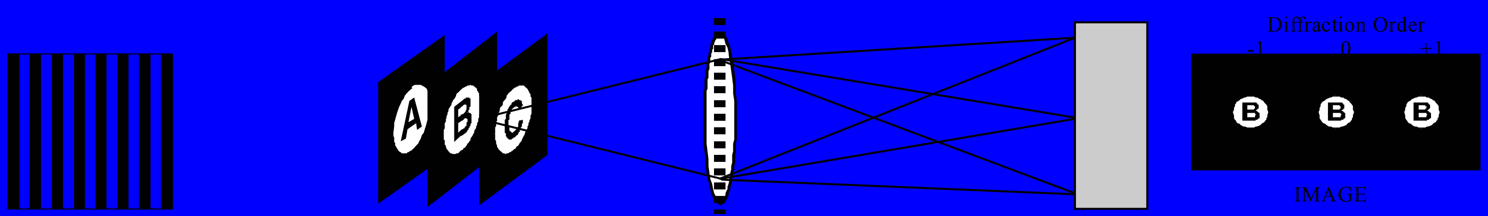
r is radius from mask centre



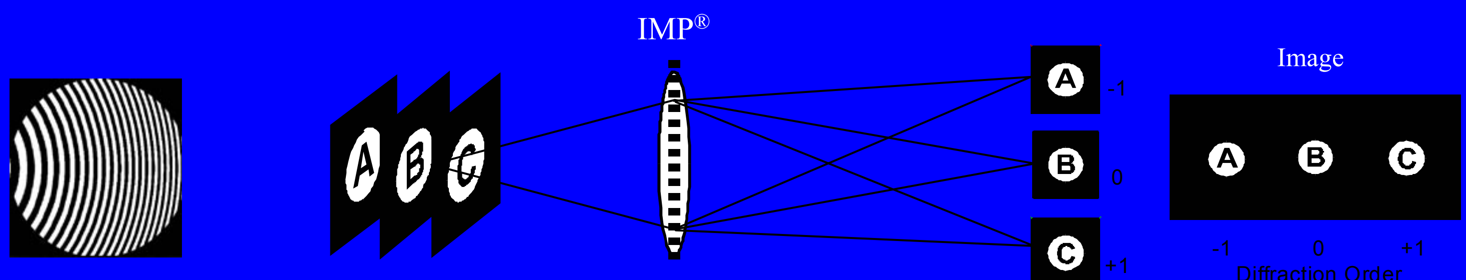
Diffractive Optics

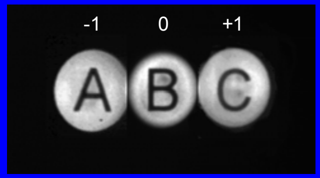
- Phase-diversity scheme needs wavefront intensity pattern on two separate planes: Scheme adopted uses IMP®s

Undistorted Grating - identical images of a single object layer in each order



Distorted Grating - images of different object layers on a single image plane

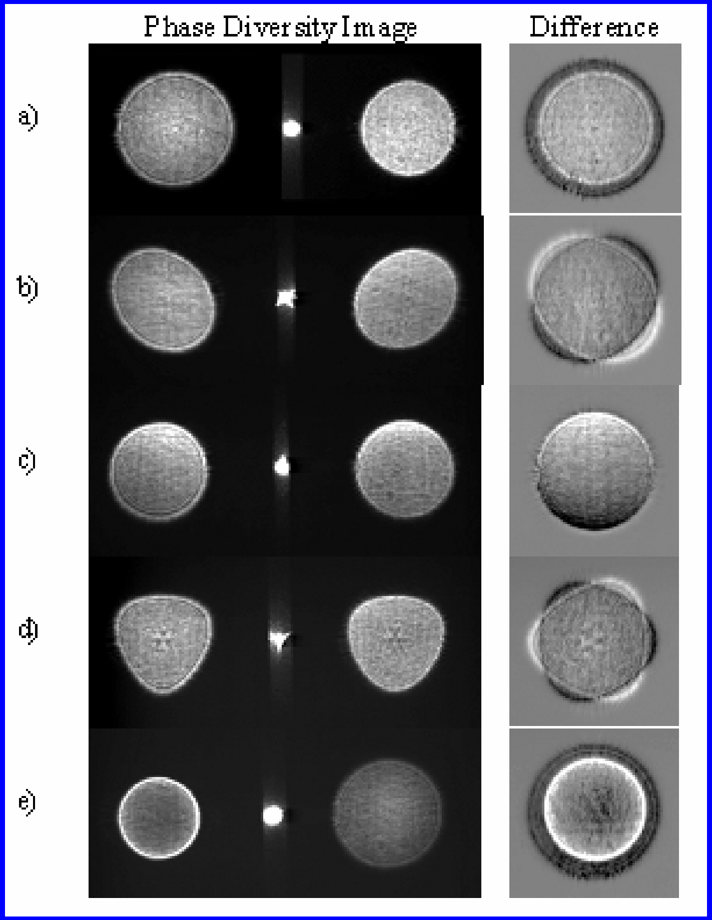




IMP® is a DERA Trademark

DERA is now QinetiQ

Examples of Data



- Some examples of the data seen at the focal plane.

- Easy to see the aberrations present in the data just by eye.

  - Defocus
  - Astigmatism
  - Coma
  - Trefoil
  - Spherical Aberration

Blanchard, P.M., et al., *Phase-diversity wave-front sensing with a distorted diffraction grating.* Applied Optics, 2000. **39**(35): p. 6649-6655.

Phase Diverse WFS

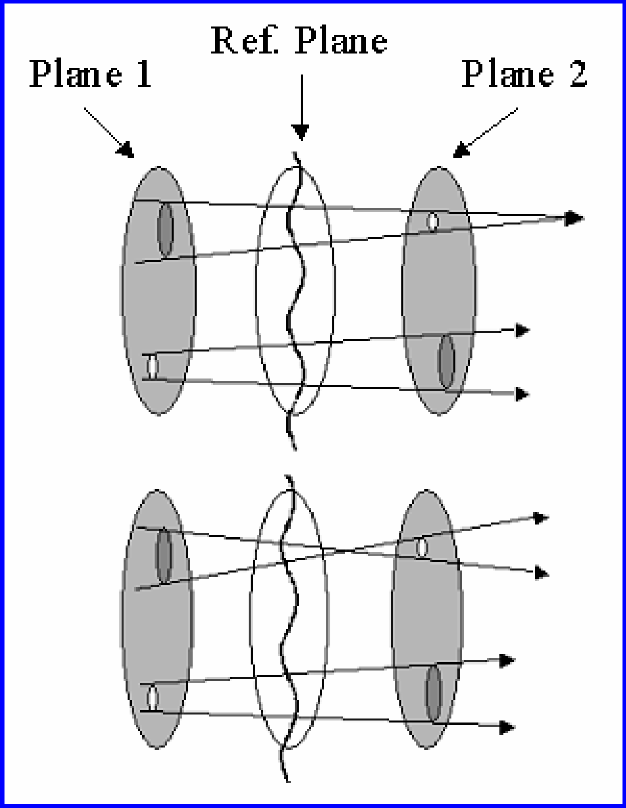


Figure 1: Two intensity planes either side of the wavefront

- DoE used to image Planes 1 & 2
- Solution of ITE gives wavefront

$$\frac{I_{\text{Plane 1}} - I_{\text{Plane 2}}}{z_1 - z_2} \approx \frac{\partial I}{\partial z}$$

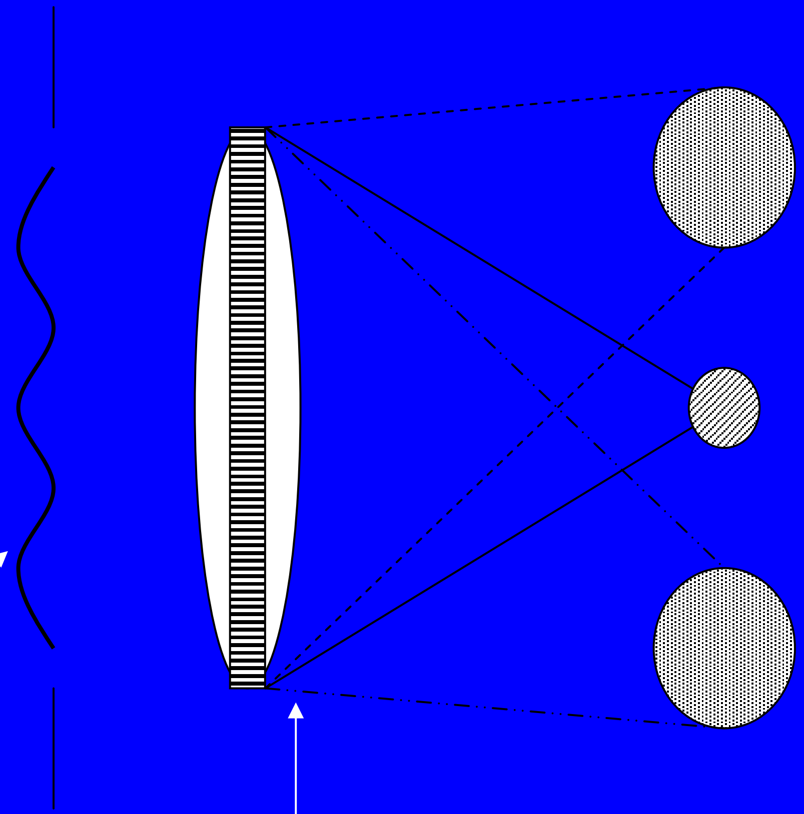
$$\Psi(r) = -k \int_R dr' G(r, r') \frac{\partial I(r')}{\partial z}$$

Restrictions

- Green's function approach using Neumann boundary conditions

> - Phase solution and its derivative must be continuous
> - Wavefront must be simply connected
> - Intensity must be constant (or small variations)
> - Deviation lead to low-pass filtered solutions

Image Plane System



$$I_{+1} = \Im\{Ae^{-i\varphi}.e^{+id\varphi}.pupil\}$$

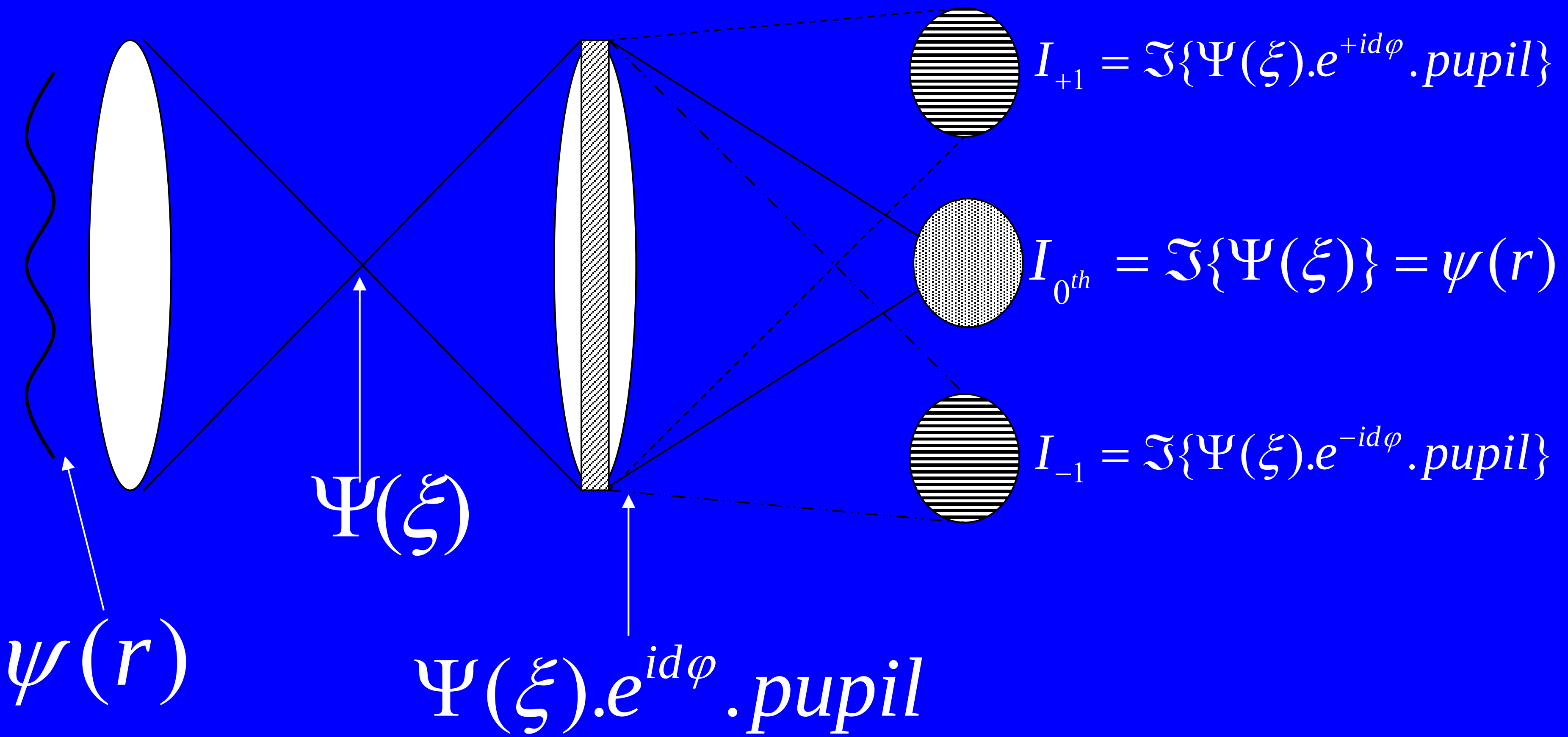
$$I_{0^{th}} = \Im\{Ae^{i\varphi}\} = \Psi(\xi)$$

$$I_{-1} = \Im\{Ae^{-i\varphi}.e^{-id\varphi}.pupil\}$$

$$\psi(r)$$

$$\psi(r).e^{id\varphi}.pupil$$

Pupil Plane System



Metrology requirements

- Wavefront may be scintillated
- Wavefront may be discontinuous
- Wavefront may be multiply connected

- Metrology of rough surfaces
- Metrology of printed circuits, integrated optics, ...
- Secondary with support structure

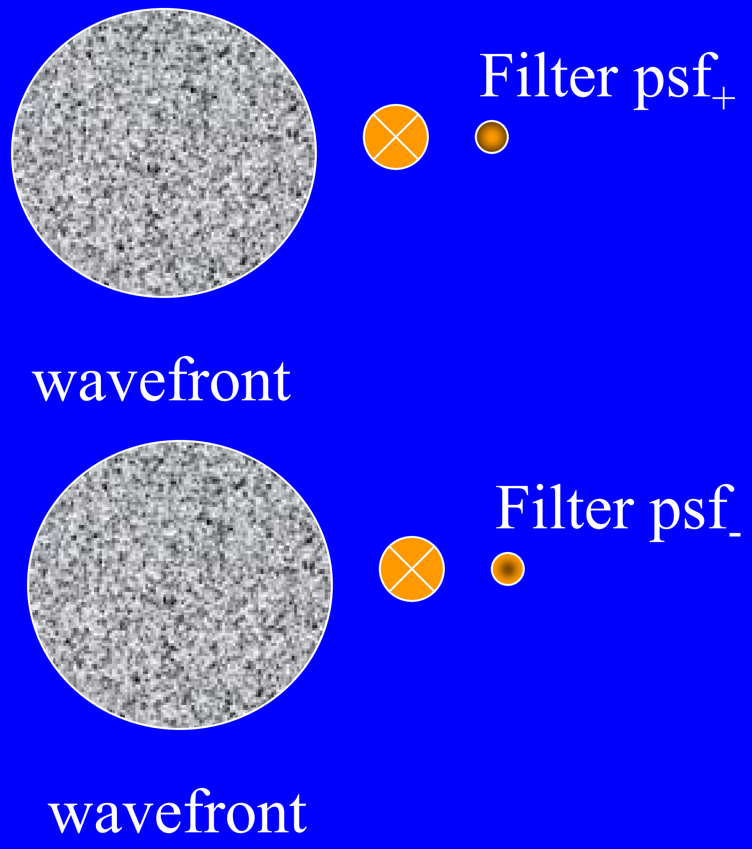
All of these are a problem for the GFS approach

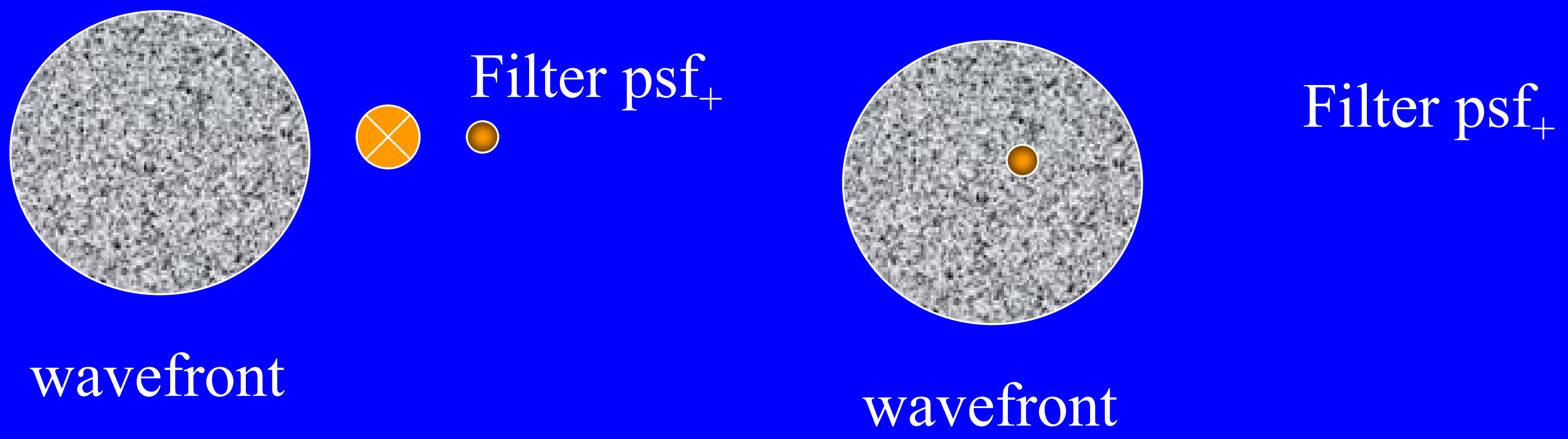
Hang on...

- Isn't another way of looking at this as the wavefront convolved with a defocused psf?
- With a defocus symmetric wrt the plane in which the wavefront is to be determined
- So what, if anything, is special about defocus?
- Is the geometric picture just a prop?
- What other convolutions might be used?

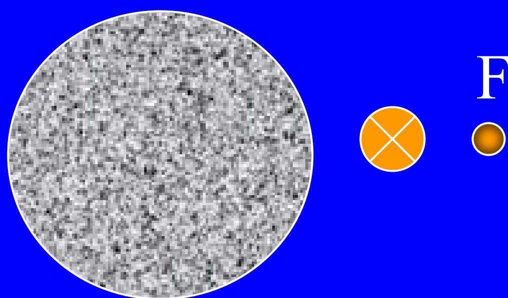
Convolution model

- Wavefront is convolved with a filter function
- In conventional phase diversity this represents propagation
- Convolution represents propagation equal distances either side of measurement plane

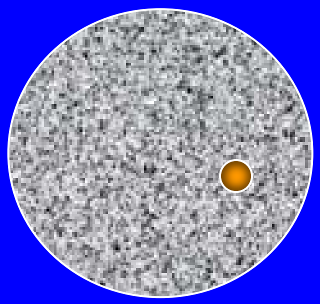



Filter psf$_+$
wavefront
Filter psf$_+$
wavefront

Filter psf$_+$

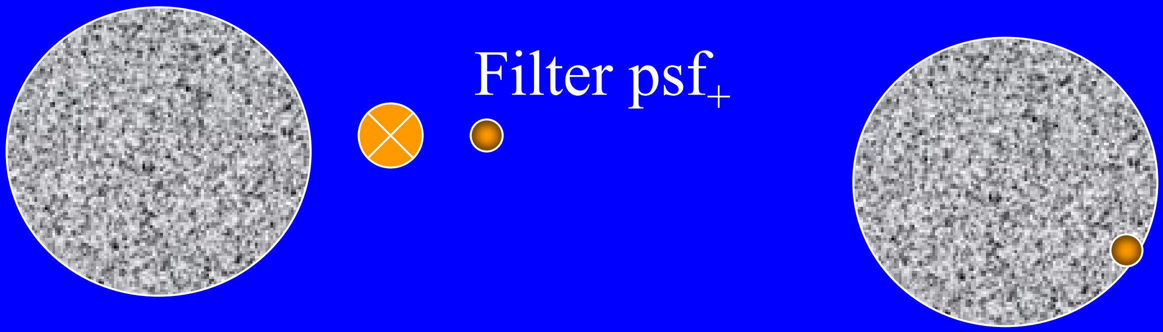


wavefront



Filter psf$_+$

wavefront



Filter psf_{+}

wavefront

Filter psf_{+}

wavefront

Null sensing

- The basic requirement for AO is a null sensor
  - It is sufficient, but not necessary, to have a metric that is zero if the wavefront is flat
  - It is desirable, but not necessary, to have a wfs that indicates the sense of any deviation from flatness

Symmetry conditions

- Required properties for aberration functions are†:
  - Filter function must be complex
  - Real and imaginary parts must have same symmetry
  - Two filter function with opposite sense can be used to provide signal after subtraction

† Campbell, Zhang, Greenaway and Restaino, Opt Lett, 29(2004)2707-2709

Metrology

- Metrology applications need numbers…
- Model test wavefront
  - Hermitian
  - Anti-Hermitian parts

- ➢ Need data inversion
- ➢ Ideally analytic
  - ➢ If not, iterations must converge quickly
- ➢ How to invert data for generalised phase diversity?

$$\frac{\mathrm{d}(r)}{2i} = \int d\xi\, \mathrm{H}(\xi)\,\mathrm{I}(\xi)e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\,\mathrm{R}(\xi')e^{ir.\xi'} - \int d\xi\, \mathrm{H}(\xi)\,\mathrm{R}(\xi)e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\,\mathrm{I}(\xi')e^{ir.\xi'}$$
$$+ \int d\xi\, \mathrm{A}(\xi)\,\mathrm{I}(\xi)e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\,\mathrm{R}(\xi')e^{ir.\xi'} - \int d\xi\, \mathrm{A}(\xi)\,\mathrm{R}(\xi)e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\,\mathrm{I}(\xi')e^{ir.\xi'}$$

Diffractive optics

- Such filters are suitable for DOEs
- Equal and opposite errors automatically produced in $\pm$ orders
- Consistent with use of defocus error

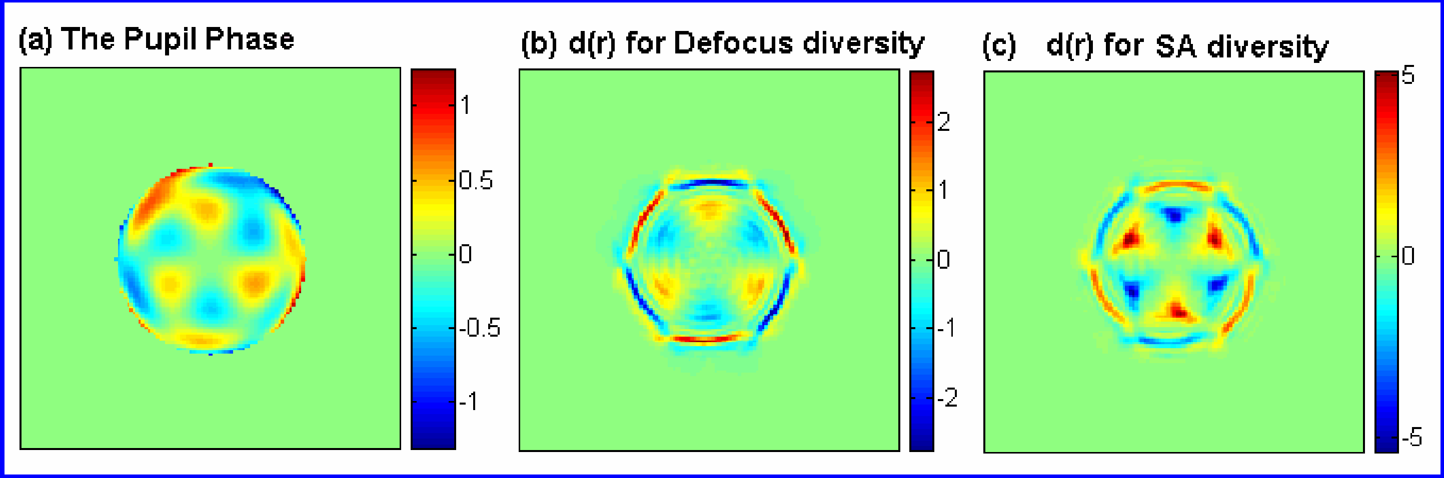
Then:

- null signal $\longrightarrow$ flat wavefront
- reversing sign of error reverses sign of signal
- Position of signal may show error location

$$\frac{\mathrm{d}(r)}{2i} = \int d\xi\,\mathrm{H}(\xi)\,\mathrm{I}(\xi)e^{-ir.\xi}\int d\xi'\,\mathrm{H}^*(\xi')\,\mathrm{R}(\xi')e^{ir.\xi'} - \int d\xi\,\mathrm{H}(\xi)\,\mathrm{R}(\xi)e^{-ir.\xi}\int d\xi'\,\mathrm{H}^*(\xi')\,\mathrm{I}(\xi')e^{ir.\xi'}$$
$$+\int d\xi\,\mathrm{A}(\xi)\,\mathrm{I}(\xi)e^{-ir.\xi}\int d\xi'\,\mathrm{A}^*(\xi')\,\mathrm{R}(\xi')e^{ir.\xi'} - \int d\xi\,\mathrm{A}(\xi)\,\mathrm{R}(\xi)e^{-ir.\xi}\int d\xi'\,\mathrm{A}^*(\xi')\,\mathrm{I}(\xi')e^{ir.\xi'}$$

Computer simulation

- Same wavefront aberration with defocus and with spherical aberration kernel
- Can produce greater signal amplitude for same signal input and same amplitude in kernel function



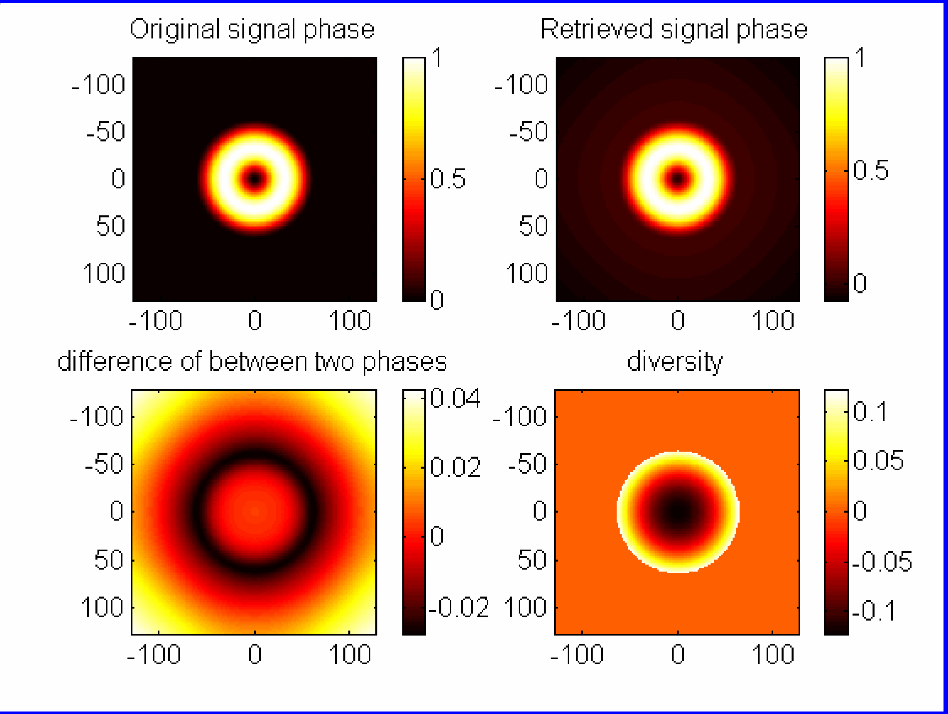
Generalised phase diversity

- Initially tested signal amplitude
- Generalised analytic solution hard
- Small-angle approximation

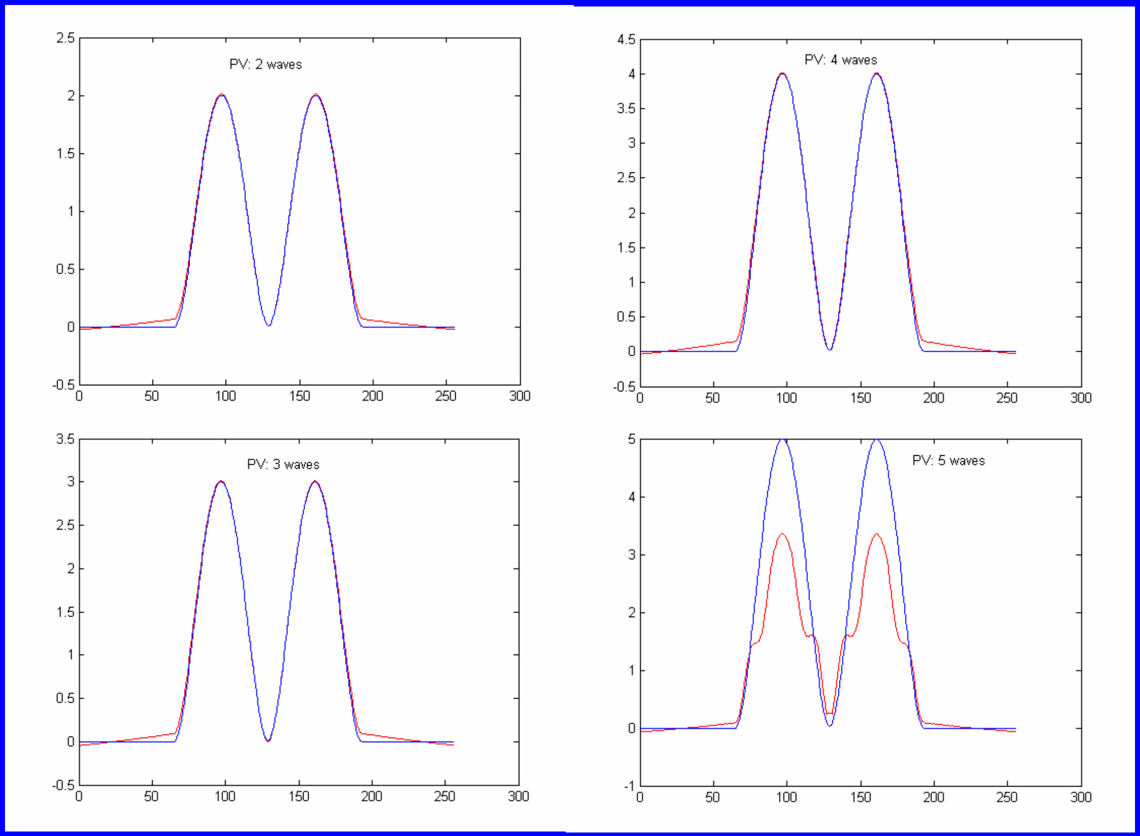
- Many suitable filters, best signal wavefront dependent
- Tried Gerchberg-Saxton - often hangs
- Successful with small signals
- Now using SAE with iterative refinement

Continuous phase...

- Explore the phase reconstruction systematically
- Continuous phase distribution ‘illuminated’ with a ‘laser beam waist’ (flat, Gaussian profile)

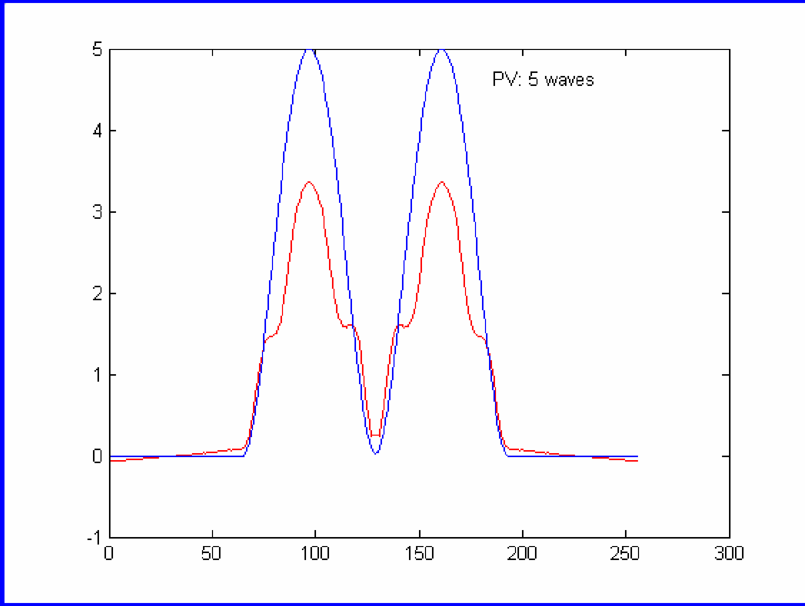


Continuous phase...



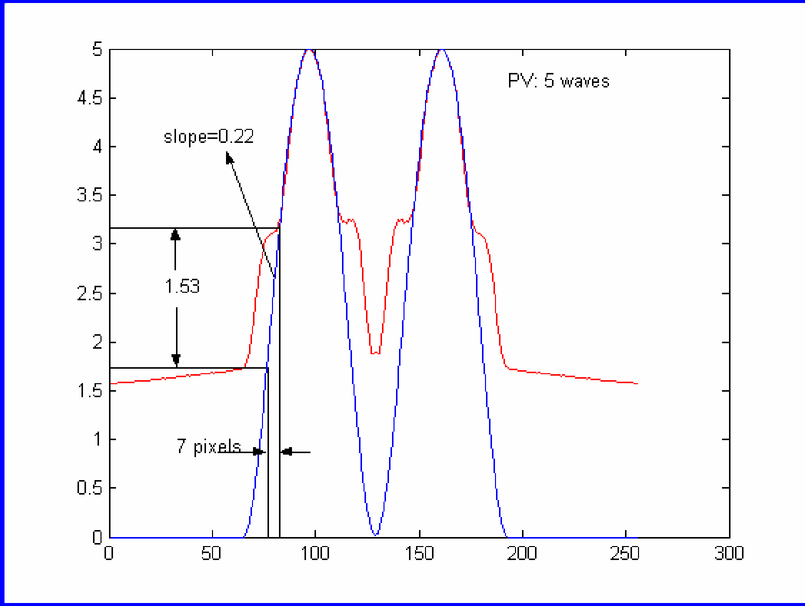
Continuous phase...

- Reconstructions seem good up to a limit, then we have sudden failure
- Note that a section fails - after this the reconstruction shape is good again
- Failure due to slope + phase swing?

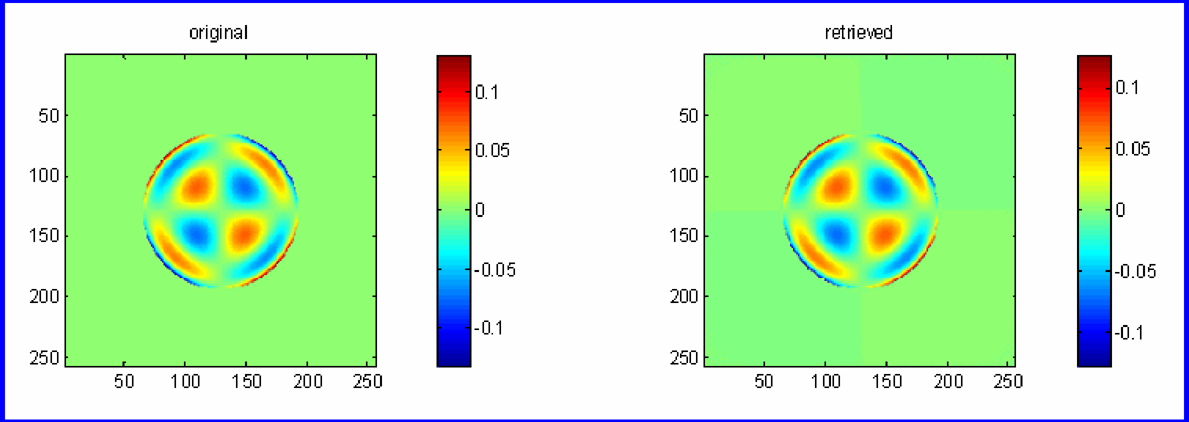


Limitations...

- We are still trying to determine how slope & amplitude combine to cause problem
- Hope to modify algorithm - but may need to modify diversity filter



SAE

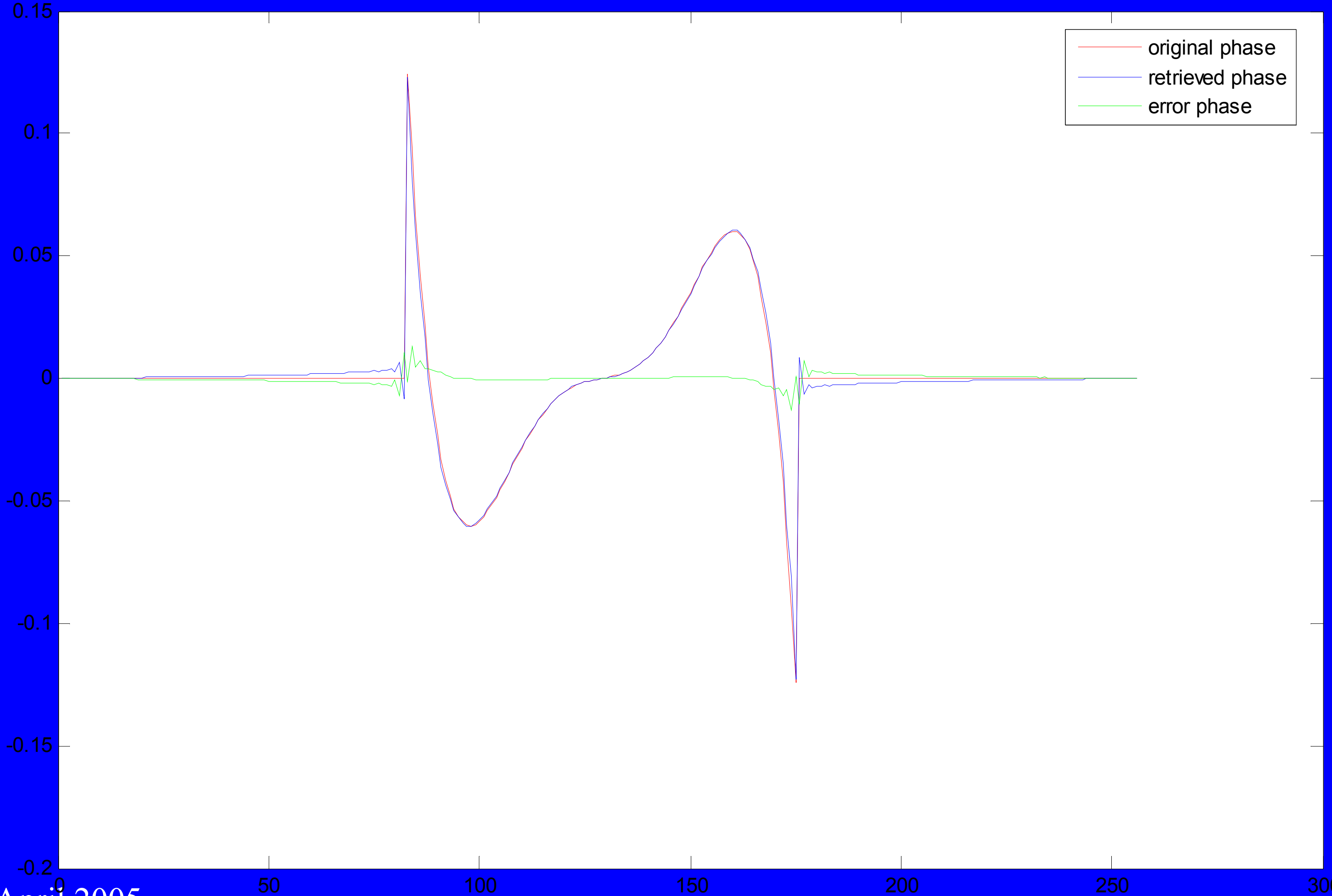


PV error of input Wavefront = 0.27λ

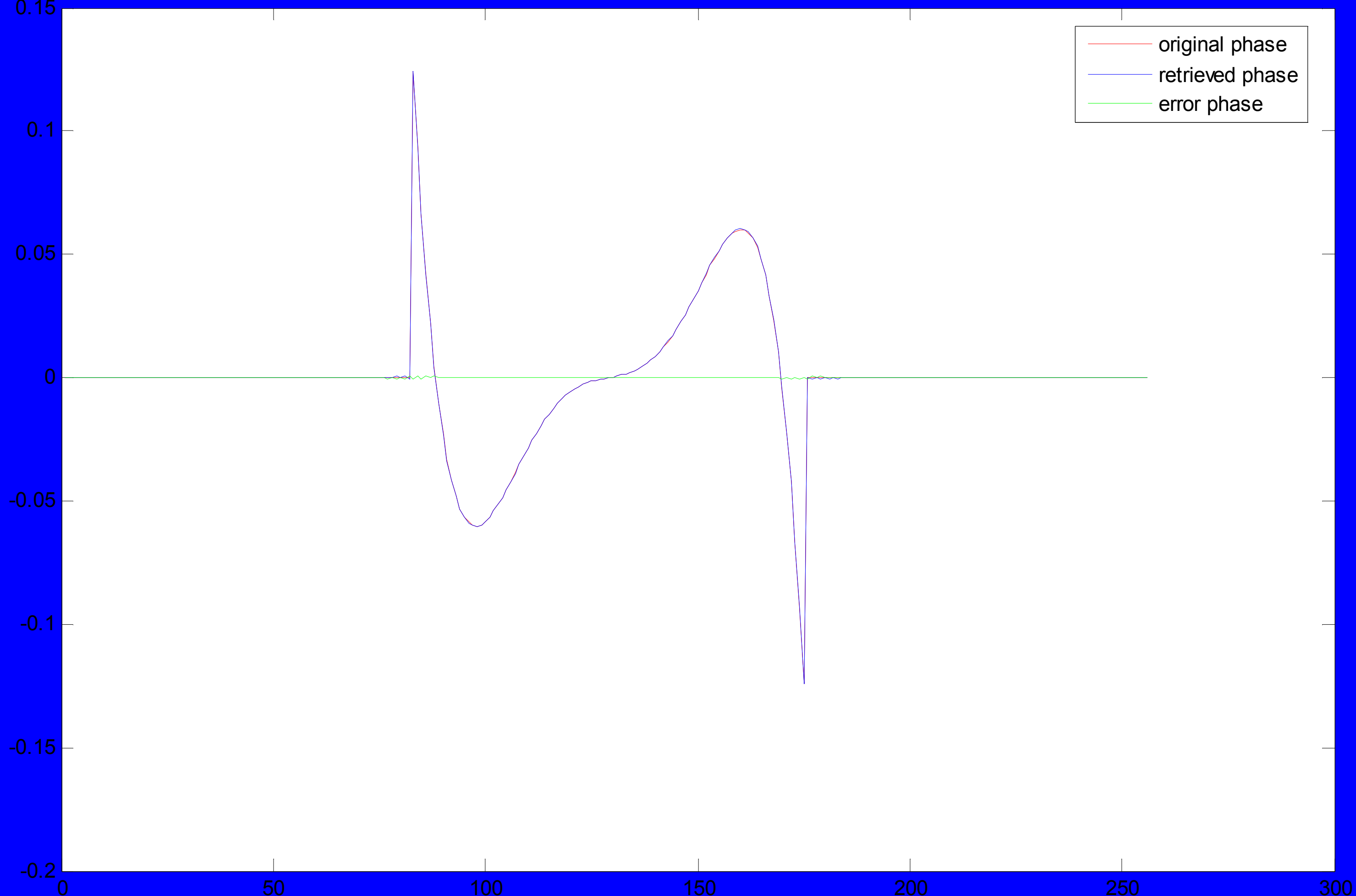
Defocus diversity Used.

- This figure shows the original input phase, and the retrieved phase calculated by the SAE algorithm. New data is computed from the solution and compared to the 'measured' data.
- An 'error' phase is calculated from the difference and used to iteratively refine the solution.

Direct application of SAE (section through 2-d sample)



After second iteration...

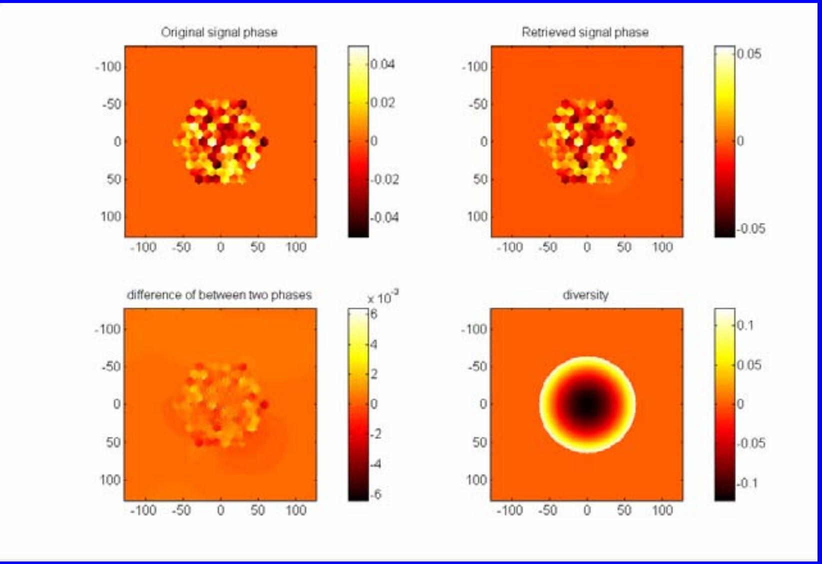


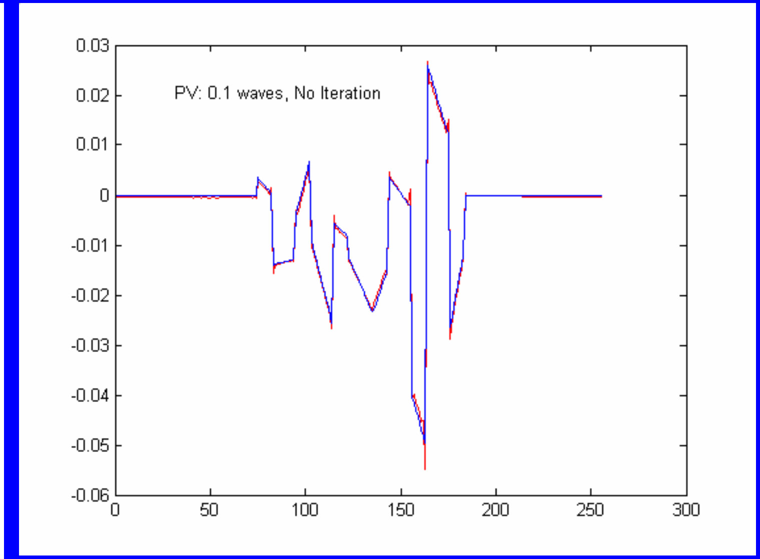
Discontinuous wavefronts

- Theory suggests signal is null for π phase jump
- Small discontinuities reconstruct easily

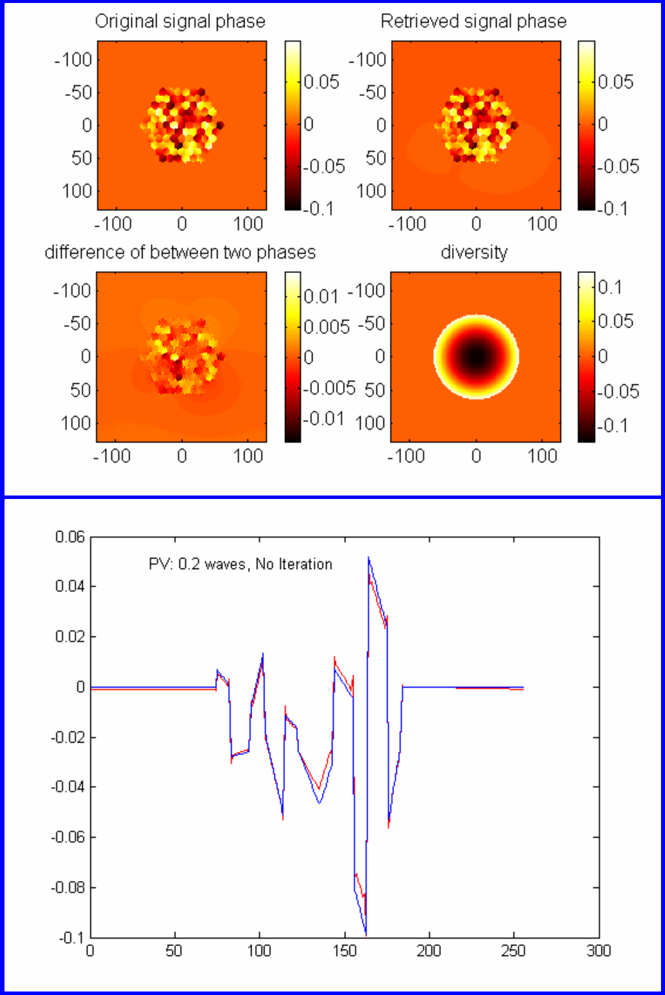
➢ Initially a surprise, but verified by simulations

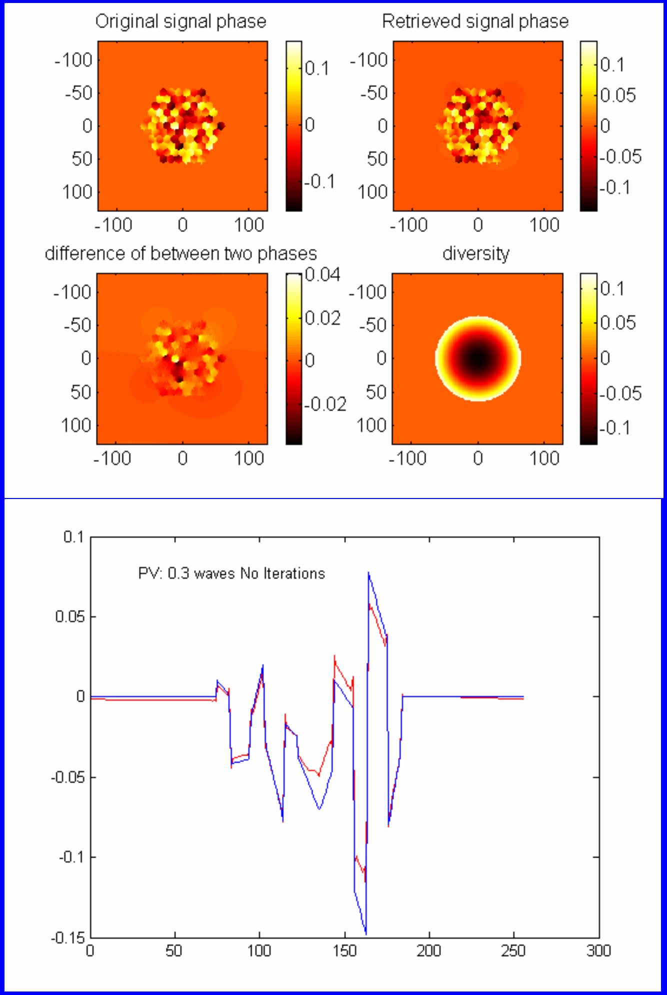
➢ 'No problem' if step is <<π/2



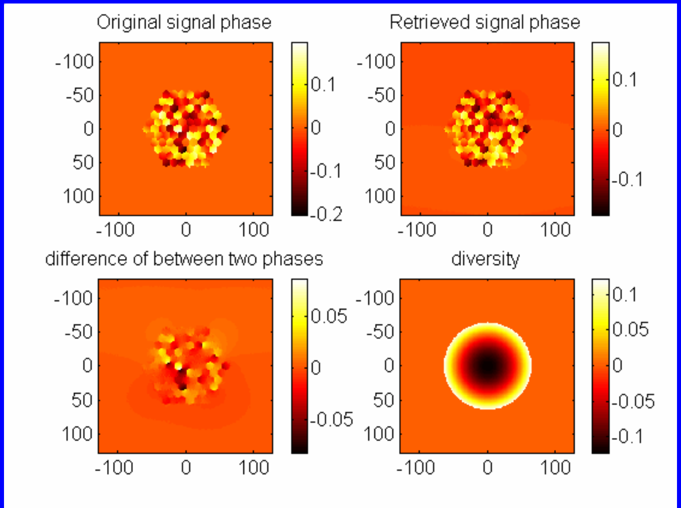


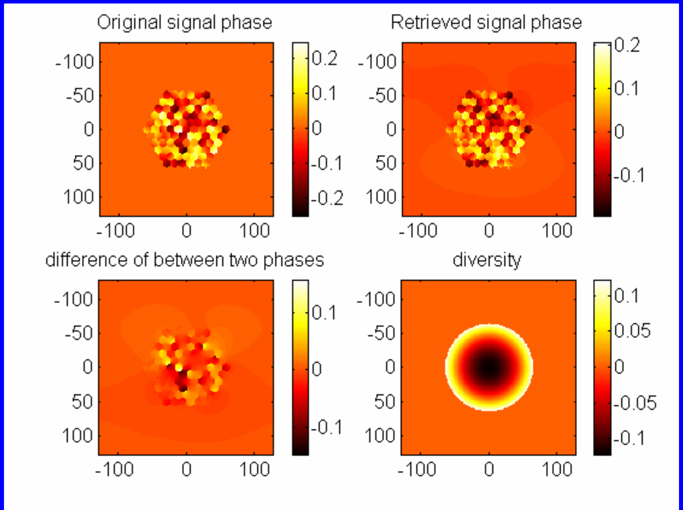
Problems with larger steps...

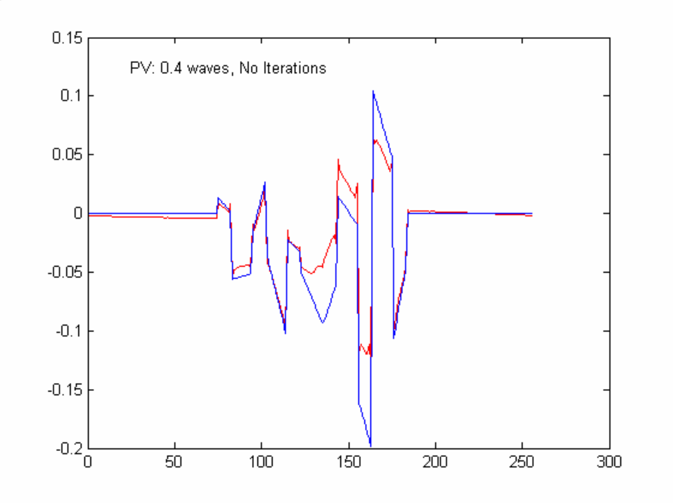


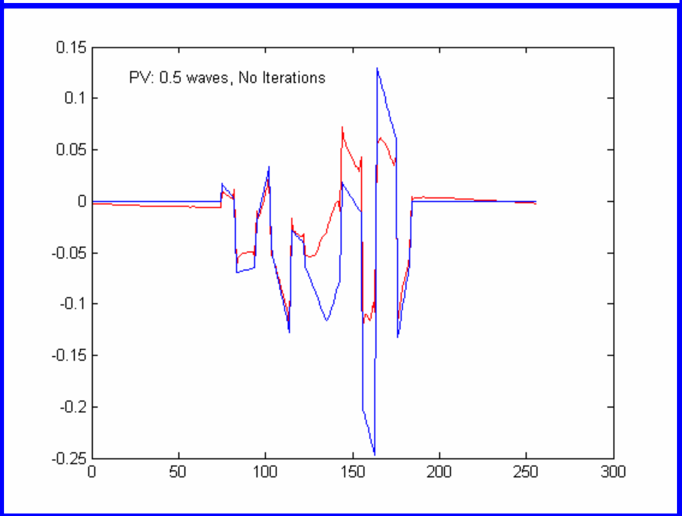


Failure with large steps









Summary

- Phase diversity can be generalised
  - Signal can be increased wrt defocus
  - Possibility to optimise filter (using statistical model of zernike modes?)
- Data inversion can be fast
  - Rapid convergence, cope with small discontinuities, speckle, multiple-connectivity?

Summary

- Completely analytic solution not yet found
    - ➢ Still considered desirable and sought for
- Is speed a problem
    - ➢ Convergence is fast and computation scales as NlogN, not N^4 (GFS)
- Is increased sensitivity useful, or is need to optimise restricting?
    - ➢ If restricting, is defocus the best general choice?
- Is boundary-value problem solved?
    - ➢ Indication that this still requires more work